UNIVERSITÀ DEGLI STUDI ROMA TRE - MATEMATICA

ESERCIZI DI AM120

A.A. 2024/2025 - ESERCITAZIONE III

Esercizio 1. Discutere il comportamento delle seguenti serie numeriche:

(1) $\displaystyle\sum_{n=1}^{+\infty}(-1)^n\frac{1}{n^\alpha}\ln\left(1+\frac{1}{n}\right) \quad (\alpha\in\mathbb{R})\,;$

(2) $\displaystyle\sum_{n=0}^{+\infty}\frac{n^2x^n}{5^n} \quad (x\in\mathbb{R})\,;$

(3) $\displaystyle\sum_{n=1}^{+\infty}\frac{\sin\frac{x}{n}}{n^x} \quad (x\in\mathbb{R})\,;$

(4) $\displaystyle\sum_{n=1}^{+\infty}\frac{1}{(\ln x)^{\ln n}} \quad (x>1)\,;$

(5) $\displaystyle\sum_{n=1}^{+\infty}\frac{n^2}{\left(x+\frac{1}{n}\right)^{\ln n}} \quad (x\geq 0)\,;$

(6) $\displaystyle\sum_{n=1}^{+\infty}\frac{x^n\ln n}{1+\sqrt{n}} \quad (x\in\mathbb{R})\,;$

Per casa.

a) $\displaystyle\sum_{n=0}^{+\infty}\frac{x+n}{1+n^3x^2} \quad (x\in\mathbb{R})\,;$

b) $\displaystyle\sum_{n=1}^{+\infty}\frac{n^x+n^2}{n^{2x}+n^3} \quad (x\in\mathbb{R})\,;$

c) $\displaystyle\sum_{n=1}^{+\infty}\frac{1}{n}\sin\left(n\sqrt{1+\frac{1}{n^2}}-n\right)\,;$

d) $\displaystyle\sum_{n=2}^{+\infty}\frac{1}{\ln(n!)}\,;$